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Comment on "Laminar Thermal Boundary Layers on Continuous Surfaces"

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IN a recent Note[1] heat-transfer rates in laminar boundary layers on continuous surfaces were discussed and compared to those of a semi-infinite flat plate. The analysis was performed numerically. As was shown in Ref. 2 this can be done analytically by an approximation yielding results which agree well with those obtained numerically. The calculation procedure is based on linearization of the boundary-layer equations, first proposed by Piercy and Preston[3] and Weyl.[4] Using a Taylor series expansion for the velocity at the wall results easily can be obtained as was shown by Schlünder[5] for the flat plate.

For constant wall temperature, the thermal boundary-layer equation in dimensionless form

$$(\partial^2\theta/\partial\eta)+\tfrac{1}{2}Pr(\partial\theta/\partial\eta)f=0 \qquad (1)$$

with boundary values for dimensionless temperature $\theta_{\eta=0}=0$, $\theta_{\eta=\infty}=1$ can be integrated, when the dimensionless stream function f is assumed to be known. As solution for the heat-transfer coefficient one obtains

$$\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0} = 1\bigg/\int_0^\infty \exp\left(-\tfrac{1}{2}Pr\int_0^\eta f\,d\eta\right)d\eta \qquad (2)$$

With two terms of a Taylor series expansion of the velocity profile $\partial f/\partial\eta$ at the wall

$$1+(\partial^2 f/\partial\eta^2)\eta+\cdots$$

Integration of Eq. (2) yields the final result

$$(\partial\theta/\partial\eta)_{\eta=0}\equiv Nu/(Re)^{1/2} \qquad (3)$$

$$=(Pr)^{1/2}\bigg/\left[(\pi)^{1/2}-\frac{2}{3}\frac{(\partial^2 f/\partial\eta^2)_{\eta=0}}{(Pr)^{1/2}}+0.738\frac{(\partial^2 f/\partial\eta^2)_{\eta=0}}{Pr}\right.$$

With dimensionless wall-shear stress

$$(\partial^2 f/\partial\eta^2)_{\eta=0}=-0.444$$

results calculated from Eq. (3) are in rather good agreement with the curve shown in Fig. 1 of Ref. (1) in the range of

$$0.1\leq Pr\leq 1000$$

Even for more complex problems, as the boundary layer behind a shock wave with vaporization and combustion closed analytical solutions of suitable accuracy are obtained by linearization[2] compared to results of an analog computer study.[6]

The advantage of a closed solution is to show general tendencies. For example from Eq. (3) the asymptotic behavior with large Prandtl number may be seen. If $Pr\to\infty$, then $Nu/(Re\cdot Pr)^{1/2}=1/(\pi^{1/2})$. The same value is valid for the semi-infinite flat plate with $Pr\to 0$ explained by the fact, that in both cases flow velocity is a constant in the region of thermal boundary layer. If, however, $Pr\to 0$ in the case of the continuous moving surface then boundary-layer theory no longer is valid for heat-transfer calculations.

The potential flow induced by the boundary-layer flow is determined with the normal velocity component at the edge of boundary layer, which is $v_E\sim 1/x^{1/2}$. The same relation holds for the plane turbulent jet, where potential flow calculations already have been performed.[7] They show that the longitudinal velocity u_E is of the same order of magnitude as the normal component. Therefore heat-transfer calculations on the assumption of $v_E\sim 1/x^{1/2}$, $u_E=0$ yielding $Nu/(RePr)^{1/2}=0.808Pr^{1/2}$ [8] seem not to be correct. The same value of heat transfer is given in Ref. (1).

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Reply by Authors to H. E. Eickhoff

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THE authors wish to thank H. E. Eickhoff for informing us of his work on continuous moving surfaces and for his interesting comments. The series expansion technique used by Eickhoff has the advantage that it leads to a better understanding of the asymptotic behavior with some sacrifice in accuracy.

Eickhoff correctly points out we have assumed in Ref. 1 that the freestream longitudinal velocity component $u_E=0$ negligible in determining the small Prandtl number heat transfer. The energy equation for the region outside the boundary layer contains the terms $u_E\,\partial T/\partial x$ and $v_E\,\partial T/\partial y$. Since the temperature gradient in the x direction is small compared to that in the y direction, the former term is small compared to the latter even

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